Spin depolarization due to beam-beam collisions

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The effect of the beam-beam interaction on spin depolarization in a proton-proton collider has been studied. The employed method is based on a matrix formalism for spin advance and for perturbed betatron particle motion in a ring. Calculations were done for a collider with one interaction point and two installed Siberian Snakes in each ring. A matrix for spin advance after an arbitrary large number of turns is found. Performed study indicates that spin depolarization due to beam-beam collisions is suppressed if the beam-beam interaction is stable and if the operation point is far enough from spin resonances. Meanwhile, in the absence of snakes or under beam-beam instability, spin is a subject of strong depolarization. Analytical estimations are confirmed by results of computer simulations. [S1063-651X(98)06507-6]

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I. INTRODUCTION

Particle colliders with polarized beams require careful control of spin depolarization. During acceleration, spin is subjected to intrinsic and imperfection resonances, resulting in depolarization. An extra source of depolarization is beam-beam collisions. Due to beam-beam interaction, particle motion become essentially nonlinear and, under some circumstances, unstable. In the present paper, the effect of beam-beam collision on spin depolarization in a proton-proton collider is studied. Betatron particle motion is defined as a linear oscillator perturbed by a nonlinear beam-beam kick. Spin rotation is described by subsequent spin matrix multiplication in a dipole magnet, in Siberian Snakes, and in the beam-beam interaction point. Analytical treatment of the problem provides a choice of the collider operation point, where depolarization is suppressed. It also indicates a zone of relatively strong depolarization.

II. SPIN MATRIX FORMALISM

Rotation of spin $\vec{S}$ of a particle with charge q, mass m, velocity $\vec{\beta}=\vec{v}/c$, and energy γ is governed by the Bargmann-Michel-Telegdi (BMT) equation [1]:

$$\frac{d\vec{S}}{dt}=\frac{q}{m\gamma}\vec{S}\times\left[(1+G\gamma)\vec{B}_{\perp}+(1+G)\vec{B}_{\parallel}+\left(G\gamma+\frac{\gamma}{1+\gamma}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right], \tag{2.1}$$

where $G=1.792\,85$ is an anomalous magnetic momentum of the proton, $\vec{E}$ is an electrical field, and $\vec{B}_{\perp}$ and $\vec{B}_{\parallel}$ are components of magnetic field, perpendicular and parallel to particle velocity, respectively,

$$\vec{B}_{\perp}=\frac{1}{v^2}(\vec{v}\times\vec{B})\times\vec{v}, \tag{2.2}$$

$$\vec{B}_{\parallel}=\frac{1}{v}(\vec{v}\cdot\vec{B})\frac{\vec{v}}{v}. \tag{2.3}$$

Particle velocity $\vec{v}$ is expanded in the orthonormal set of curvilinear coordinate $(\vec{x},\vec{y},\vec{z})$ as follows:

$$\vec{v}=\dot{z}\left[x'\vec{x}+y'\vec{y}+\left(1+\frac{x}{\rho}\right)\vec{z}\right], \tag{2.4}$$

where ρ is a curvature radius of the reference coordinate system, the dot means a derivative over time, and the prime means a derivative over longitudinal coordinate. Change of the independent variable in Eq. (2.1) from t for z gives

$$\frac{d\vec{S}}{dt}=\frac{d\vec{S}}{dz}\frac{dz}{dt}=\frac{d\vec{S}}{dz}\frac{v}{\left[\left(1+\frac{x}{\rho}\right)^2+x'^2+y'^2\right]^{1/2}}. \tag{2.5}$$

Calculation of magnetic field components (2.2) and (2.3) results in the following expressions:

$$\vec{B}_{\perp}=\left[x'^2+y'^2+\left(1+\frac{x}{\rho}\right)^2\right]^{-1}\left(\left\{\left[\left(1+\frac{x}{\rho}\right)^2+y'^2\right]B_x-x'y'B_y-x'B_z\left(1+\frac{x}{\rho}\right)\right\}\vec{x}+\left\{-x'y'B_x+\left[\left(1+\frac{x}{\rho}\right)^2+x'^2\right]B_y-y'B_z\left(1+\frac{x}{\rho}\right)\right\}\vec{y}+\left[-x'B_x\left(1+\frac{x}{\rho}\right)-y'B_y\left(1+\frac{x}{\rho}\right)+(x'^2+y'^2)B_z\right]\vec{z}\right), \tag{2.6}$$

$$\vec{B}_{\|}=\left[x'^2+y'^2+\left(1+\frac{x}{\rho}\right)^2\right]^{-1}\left[x'B_x+y'B_y+\left(1+\frac{x}{\rho}\right)B_z\right]\left[x'\vec{x}+y'\vec{y}+\left(1+\frac{x}{\rho}\right)\vec{z}\right]. \tag{2.7}$$

The vector product of the electric field and the particle velocity gives

$$\vec{E}\times\vec{v}=\dot{z}\left\{\left[E_y\left(1+\frac{x}{\rho}\right)-E_z y'\right]\vec{x}+\left[E_z x'-\left(1+\frac{x}{\rho}\right)E_x\right]\vec{y}+[y'E_x-E_y x']\vec{z}\right\}. \tag{2.8}$$

Combining all terms, the BMT equation now can be written as

$$\frac{d\vec{S}}{dz}=\vec{S}\times\vec{P} \tag{2.9}$$

or

$$\frac{dS_x}{dz}=S_y P_z-S_z P_y,$$

$$\frac{dS_y}{dz}=S_z P_x-S_x P_z, \tag{2.10}$$

$$\frac{dS_z}{dz}=S_x P_y-S_y P_x,$$

where the vector $\vec{P}=(P_x,P_y,P_z)$ is given by the terms up to first order by the following expressions:

$$P_x=\frac{q}{m\gamma v}\left[(1+G\gamma)(B_x-x'B_z)+(1+G)x'B_z+\frac{v}{c^2}\left(\frac{\gamma}{1+\gamma}+G\gamma\right)(E_y-y'E_z)\right], \tag{2.11}$$

$$P_y=\frac{q}{m\gamma v}\left[(1+G\gamma)(B_y-y'B_z)+(1+G)y'B_z+\frac{v}{c^2}\left(\frac{\gamma}{1+\gamma}+G\gamma\right)(x'E_z-E_x)\right], \tag{2.12}$$

$$P_z=\frac{q}{m\gamma v}\left[(1+G\gamma)(-x'B_x-y'B_y)+(1+G)(x'B_x+B_z+y'B_y)+\frac{v}{c^2}\left(\frac{\gamma}{1+\gamma}+G\gamma\right)(y'E_x-E_y x')\right]. \tag{2.13}$$

To derive the matrix of spin rotation, let us assume that the vector $\vec{P}$ is a constant at the infinitesimal distance δz. The second derivative of the spin vector is given by

$$\frac{d^2S_x}{dz^2}=S_zP_xP_z-S_x(P_z^2+P_y^2)+S_yP_xP_y,$$

$$\frac{d^2S_y}{dz^2}=S_xP_xP_y-S_y(P_x^2+P_z^2)+S_zP_yP_z, \tag{2.14}$$

$$\frac{d^2S_z}{dz^2}=S_yP_yP_z-S_z(P_y^2+P_x^2)+S_xP_zP_x,$$

Taking the third derivative of spin vector, Eqs. (2.10) are reduced to the third-order differential equations:

$$S_x'''+P_0^2S_x'=0,$$

$$S_y'''+P_0^2S_y'=0, \tag{2.15}$$

$$S_z'''+P_0^2S_z'=0,$$

$$P_0^2=P_x^2+P_y^2+P_z^2. \tag{2.16}$$

A general solution to the problem (2.15) can be written in the form

$$S_x=C_{x1}+C_{x2}\cos(P_0\delta z)+C_{x3}\sin(P_0\delta z),$$

$$S_y=C_{y1}+C_{y2}\cos(P_0\delta z)+C_{y3}\sin(P_0\delta z), \tag{2.17}$$

$$S_z=C_{z1}+C_{z2}\cos(P_0\delta z)+C_{z3}\sin(P_0\delta z),$$

where constants C_{ij}, $i=(x,y,z)$, $j=(1,2,3)$ depend on initial conditions.

Let us express constants in Eqs. (2.17) through initial values of spin and its derivatives. Assuming in Eq. (2.17) $\delta z=0$, the initial value of spin vector $\vec{S}_0=(S_{x0},S_{y0},S_{z0})$ is given by

$$S_{x0}=C_{x1}+C_{x2},$$

$$S_{y0}=C_{y1}+C_{y2}, \tag{2.18}$$

$$S_{z0}=C_{z1}+C_{z2}.$$

From Eqs. (2.17), initial values of the first $\vec{S}_0'=(S_{x0}',S_{y0}',S_{z0}')$ and of the second $\vec{S}_0''=(S_{x0}'',S_{y0}'',S_{z0}'')$ order derivatives of spin vector are

$$S_{x0}'=C_{x3}P_0,$$

$$S_{y0}'=C_{y3}P_0, \tag{2.19}$$

$$S_{z0}'=C_{z3}P_0,$$

$$S_{x0}''=-C_{x2}P_0^2,$$

$$S_{y0}''=-C_{y2}P_0^2, \tag{2.20}$$

$$S_{z0}''=-C_{z2}P_0^2.$$

Combining Eqs. (2.17)–(2.20), solution for spin advance at the distance δz can be written as follows:

$$\vec{S}=\vec{S}_0+\frac{\vec{S}_0'}{P_0}\sin(P_0\delta z)+\frac{\vec{S}_0''}{P_0^2}[1-\cos(P_0\delta z)]. \quad (2.21)$$

Substitution of Eqs. (2.10) and (2.14) into Eq. (2.21) gives the following matrix of spin rotation at the distance δz [2]:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1-a(B^2+C^2) & ABa+Cb & ACa-Bb \\ ABa-Cb & 1-a(A^2+C^2) & BCa+Ab \\ ACa+Bb & BCa-Ab & 1-a(A^2+B^2) \end{vmatrix} \times \begin{vmatrix} S_{x,0} \\ S_{y,0} \\ S_{z,0} \end{vmatrix}, \quad (2.22)$$

$$A=\frac{P_x}{P_0}, \quad B=\frac{P_y}{P_0}, \quad C=\frac{P_z}{P_0}, \quad (2.23)$$

$$a=1-\cos\varphi, \quad b=\sin\varphi, \quad \varphi=P_0\delta z. \quad (2.24)$$

Matrix (2.22) can be used for calculation of spin rotation in an arbitrary electromagnetic field, assuming the field is constant at the distance δz. Below, matrix (2.22) will be applied for the calculation of spin advance in a bending magnet and in a beam-beam interaction point.

III. MODEL OF COLLIDER WITH POLARIZED PARTICLES

A. Particle betatron motion

Let us consider a collider ring with two installed Siberian Snakes. We use a two-dimensional particle model in coordinates $(x,p_x=\beta_x^*(dx/dz))$, $(y,p_y=\beta_y^*(dy/dz))$, where β_x^*, β_y^* are beta functions of the ring. Particle motion between subsequent collisions combines linear matrix transformation, perturbed by beam-beam interaction:

$$\begin{pmatrix} x_{n+1} \\ p_{x,n+1} \\ y_{n+1} \\ p_{y,n+1} \end{pmatrix} = \begin{pmatrix} \cos\overline{\theta_x} & \sin\overline{\theta_x} & 0 & 0 \\ -\sin\overline{\theta_x} & \cos\overline{\theta_x} & 0 & 0 \\ 0 & 0 & \cos\overline{\theta_y} & \sin\overline{\theta_y} \\ 0 & 0 & -\sin\overline{\theta_y} & \cos\overline{\theta_y} \end{pmatrix} \times \begin{pmatrix} x_n \\ p_{x,n}+\Delta p_{x,n} \\ y_n \\ p_{y,n}+\Delta p_{y,n} \end{pmatrix}, \quad (3.1)$$

where $\overline{\theta_x}=2\pi Q_x$ and $\overline{\theta_y}=2\pi Q_y$ are betatron angles and Q_x and Q_y are betatron tunes. Beam-beam kicks $\Delta p_{x,n}$, $\Delta p_{y,n}$ are expressed as a result of an interaction of particles with opposite beam with the Gaussian distribution function

$$\Delta p_{x,n}=4\pi\xi x_n\frac{1-\exp(-r_n^2/2\sigma_n^2)}{(r_n^2/2\sigma_n^2)}, \quad (3.2)$$

and similar for $\Delta p_{y,n}$. Parameter ξ is a beam-beam parameter, which characterizes the strength of the beam-beam interaction,

$$\xi=\frac{Nr_0\beta^*}{4\pi\sigma^2\gamma}, \quad (3.3)$$

where N is a number of particles per bunch, $r_0=q^2/(4\pi\epsilon_0 mc^2)$ is a classical particle radius, and σ is a transverse standard deviation of the opposite beam size.

B. Spin matrix

Rotation of spin vector $\vec{S}=(S_x,S_y,S_z)$ is described by a subsequent matrix transformation in a lattice arc, in Siberian Snakes, and in an interaction point.

1. Dipole magnet

Spin rotation in an ideal lattice arc is described as a spin precession in a dipole magnet with bending angle v. Assume that the field of the dipole magnet has only one vertical component:

$$B_x=0, \quad B_z=0, \quad B_y=B. \quad (3.4)$$

Therefore, components of vector $\vec{P}$, Eqs. (2.11)–(2.13), and corresponding matrix coefficients, Eqs. (2.23) and (2.24), are given by

$$P_x=0, \quad P_y=\frac{(1+G\gamma)}{\rho}, \quad P_z=0, \quad A=0, \quad B=1, \quad C=0, \quad (3.5)$$

$$P_0\delta z=\frac{(1+G\gamma)}{\rho}\delta z=(1+G\gamma)v. \quad (3.6)$$

The matrix of spin rotation in the dipole magnet is [3]

$$D_v=\begin{vmatrix} \cos(P_0\delta z) & 0 & -\sin(P_0\delta z) \\ 0 & 1 & 0 \\ \sin(P_0\delta z) & 0 & \cos(P_0\delta z) \end{vmatrix}. \quad (3.7)$$

2. Siberian Snakes

Siberian Snakes rotate any spin vector by angle π around axis [4]. Two types of snakes are used, which matrixes are given by

$$S_1=\begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}, \quad (3.8)$$

$$S_2=\begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix}. \quad (3.9)$$

3. Interaction point

Spin advance after crossing an interaction point is described by matrix (2.22), where δz is an interaction distance, defined below. Vector $\vec{P}$, Eqs. (2.11)–(2.13), in the case of a head-on beam-beam collision, is as follows:

$$P_x=\frac{1}{B\rho}\left((1+G\gamma)B_x+\left(G\gamma+\frac{\gamma}{1+\gamma}\right)\frac{\beta E_y}{c}\right), \quad (3.10)$$

$$P_y=\frac{1}{B\rho}\left[(1+G\gamma)B_y-\left(G\gamma+\frac{\gamma}{1+\gamma}\right)\frac{\beta E_x}{c}\right], \quad (3.11)$$

$$P_z=0, \quad (3.12)$$

where small terms x',y' are neglected, $B\rho=mc\beta\gamma/q$ is a rigidity of particles, $\vec{E}=(E_x,E_y,0)$ is an electrical field, and $\vec{B}=(B_x,B_y,0)$ is a magnetic field of the opposite bunch. Due to Lorentz transformations, components of electromagnetic field of the opposite bunch are connected via relationships

$$B_x=\beta\frac{E_y}{c}, \quad B_y=-\beta\frac{E_x}{c}. \quad (3.13)$$

Assuming, that interacted particles are ultrarelativistic $\beta\approx 1$, $\gamma\gg 1$, the vector $\vec{P}$ is simplified,

$$P_x=\frac{qE_y}{mc^2\gamma}\left[(1+G\gamma)+\left(G\gamma+\frac{\gamma}{1+\gamma}\right)\right]\approx 2G\frac{qE_y}{mc^2}, \quad (3.14)$$

$$P_y=\frac{qE_x}{mc^2\gamma}\left[-(1+G\gamma)-\left(G\gamma+\frac{\gamma}{1+\gamma}\right)\right]\approx -2G\frac{qE_x}{mc^2}. \quad (3.15)$$

Let us express the matrix parameter φ, Eq. (2.24), via the beam-beam parameter ξ. Electrostatic field of the opposite round Gaussian bunch with length l and peak current $I=qN\beta c/l$ is

$$E_r=\frac{qN}{2\pi\varepsilon_0 lr}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right]=\frac{I}{2\pi\varepsilon_0\beta cr}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right], \quad (3.16)$$

$$E_x=E_r\frac{x}{r}, \quad E_y=E_r\frac{y}{r}. \quad (3.17)$$

Substitution of the expression of the electrostatic field into Eqs. (3.14) and (3.15) gives the expressions for vector $\vec{P}$,

$$P_x=4G\frac{I}{I_c}\frac{y}{r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right], \quad (3.18)$$

$$P_y=-4G\frac{I}{I_c}\frac{x}{r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right], \quad (3.19)$$

where $I_c=4\pi\varepsilon_0 mc^3/q=(A/Z)3.13\times 10^7$ A is a characteristic value of the beam current. The beam-beam parameter ξ in Eq. (3.3) can be rewritten as follows:

$$\xi=\frac{\beta^*}{4\pi}\frac{I}{I_c}\frac{l}{\gamma\sigma^2}. \quad (3.20)$$

To define the interaction distance δz, let us suppose that at the time moment $t=0$ the test particle enters the opposite bunch (see Fig. 1). The equation of motion of the test particle is $z_1=v_1 t$. The equation of motion of the right edge of the bunch is $z_2=l-v_2 t$. The test particle will leave the opposite bunch when $z_1=z_2$, or after the time interval $t=l/(v_1+v_2)$.

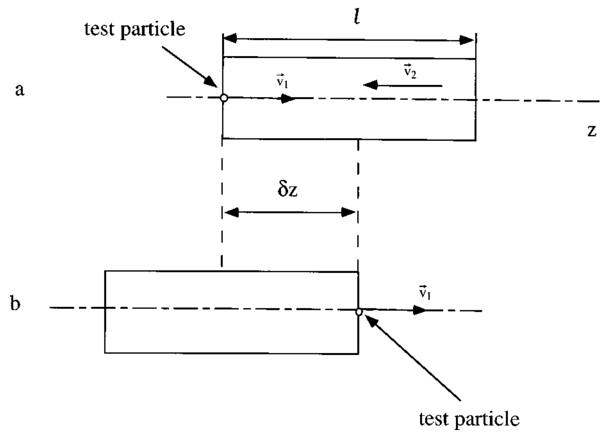

FIG. 1. Position of the test particle with respect to the opposite bunch: (a) before interaction; (b) after interaction.

The coordinate of the test particle at this moment, $z_1=v_1 t$, is equal to the interaction distance δz:

$$\delta z=v_1 t=l\frac{v_1}{v_1+v_2}=\frac{l}{2}. \quad (3.21)$$

Taking into account Eqs. (3.18) and (3.19), the parameters of the spin matrix $P_x\delta z, P_y\delta z$ can be expressed as follows:

$$P_x\delta z=P_x\frac{l}{2}=4\pi G\gamma\xi\frac{y}{\beta^*}\left[\frac{1-\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\left(\frac{r^2}{2\sigma^2}\right)}\right], \quad (3.22)$$

$$P_y\delta z=P_y\frac{l}{2}=-4\pi G\gamma\xi\frac{x}{\beta^*}\left[\frac{1-\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\left(\frac{r^2}{2\sigma^2}\right)}\right]. \quad (3.23)$$

Finally, the parameter φ is given by

$$\varphi=\sqrt{(P_x\delta z)^2+(P_y\delta z)^2}=4\pi G\gamma\xi\frac{r}{\beta^*}\left[\frac{1-\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\left(\frac{r^2}{2\sigma^2}\right)}\right]. \quad (3.24)$$

The parameter φ is typically much smaller than 2π, which gives us the possibility of simplifying the matrix of spin rotation in the interaction point and to provide an analytical treatment of the problem (see the next section).

TABLE I. Parameters of the numerical model.

Number of modeling particles, N	5000
Number of turns	10^6
CPU time (for VAX Alpha)	5 h

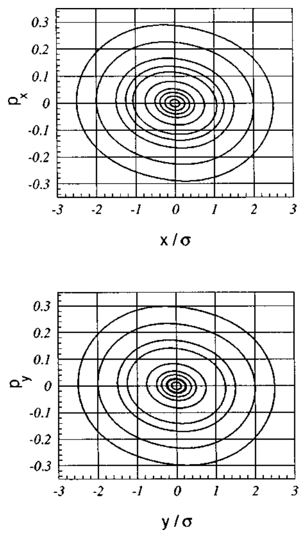

FIG. 2. Stable particle trajectories in phase space in the presence of a beam-beam interaction without noise.

The model developed in this section was incorporated into the numerical code BEAMPATH [5]. Typical parameters of the numerical model are summarized in Table I.

IV. ANALYTICAL TREATMENT OF SPIN DEPOLARIZATION

A. Simplified spin matrix in the interaction point

To make an analytical treatment of spin depolarization, let us simplify the suggested model. Consider a collider with two Siberian Snakes and one interaction point. The matrix of spin advance after one revolution in the ring between the beam-beam interaction is

$$M_{\rm ring}=D_{\pi/2}S_2D_{\pi}S_1D_{\pi/2}=\begin{vmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{vmatrix}. \qquad (4.1)$$

Suppose that betatron angles in the x and y directions are equal to each other, $\theta_x=\theta_y=\theta$. We consider particle motion far enough from low order resonances, therefore particle trajectory can be expressed as a linear oscillator with perturbed betatron tune θ:

$$x=r\cos(n\theta+\Psi),\quad y=r\sin(n\theta+\Psi),\quad \theta=\overline{\theta}+\Delta\theta, \qquad (4.2)$$

where Ψ is an initial phase of betatron particle oscillations and $\Delta\theta\ll 2\pi$ is a tune perturbation due to beam-beam collisions. In Fig. 2, an example of particle trajectories in the presence of a stable beam-beam interaction is given. Particle trajectories in phase space are slightly deformed ellipses. In this case, beam envelopes and beam emittances are also stable (see Fig. 3). Beam-beam instability and its effect on spin depolarization will be considered in Sec. V.

Parameters A and B, Eq. (2.23), at the interaction point can be expressed as follows:

$$A=\frac{P_x}{P_0}=\frac{y}{r}=\sin(n\theta+\Psi), \qquad (4.3)$$

$$B=\frac{P_y}{P_0}=-\frac{x}{r}=-\cos(n\theta+\Psi). \qquad (4.4)$$

Let us take into account that the parameter φ is small:

$$\varphi=P_0\delta z=4\pi G\gamma\xi\frac{r}{\beta^*}\left(1-\frac{r^2}{4\sigma^2}+\cdots\right)\ll 2\pi. \qquad (4.5)$$

Hence, the matrix parameters a and b are as follows:

$$a=1-\cos\varphi\approx\frac{\varphi^2}{2},\quad b=\sin\varphi\approx\varphi. \qquad (4.6)$$

Finally, the matrix of spin advance of a particle in an interaction point at the nth turn is given by

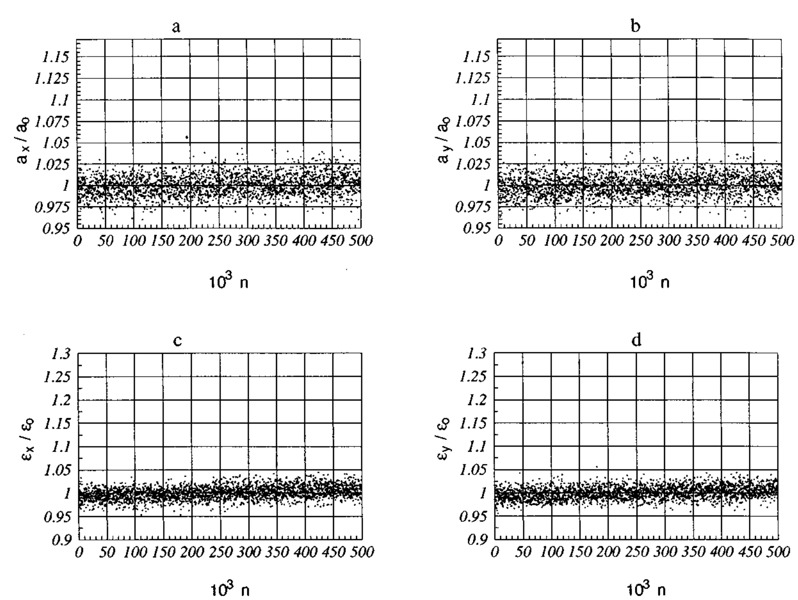

FIG. 3. (a) and (b) Beam envelopes; (c) and (d) beam emittances in the presence of a stable beam-beam interaction.

$$M_{b\text{-}b}(n)=\begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2(n\theta+\Psi) & -\frac{\varphi^2}{4}\sin 2(n\theta+\Psi) & \varphi\cos(n\theta+\Psi)\\ -\frac{\varphi^2}{4}\sin 2(n\theta+\Psi) & 1-\frac{\varphi^2}{2}\sin^2(n\theta+\Psi) & \varphi\sin(n\theta+\Psi)\\ -\varphi\cos(n\theta+\Psi) & -\varphi\sin(n\theta+\Psi) & 1-\frac{\varphi^2}{2}\end{vmatrix}. \quad (4.7)$$

B. Spin matrix after an arbitrary number of turns

Now let us derive a matrix of spin advance after an arbitrary number of turns. Due to the small value of the parameter φ, we will leave in the resulting matrix only terms proportional to φ and φ^2, while neglecting terms with φ^3, φ^4, and higher order.

Suppose the initial position of the particles is just before the interaction point. After the interaction point, the matrix of spin advance is the matrix (4.7), where $n=0$:

$$M_{b\text{-}b}(1)=\begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2\Psi & -\frac{\varphi^2}{4}\sin 2\Psi & \varphi\cos\Psi\\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi\\ -\varphi\cos\Psi & -\varphi\sin\Psi & 1-\frac{\varphi^2}{2}\end{vmatrix}. \quad (4.8)$$

After the interaction point, particles perform one revolution in the ring and the spin matrix after the first turn, $M_{1/0}$, is a product of matrixes (4.1) and (4.8):

$$M_{1/0}=\begin{vmatrix}-1&0&0\\0&1&0\\0&0&-1\end{vmatrix}\begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2\Psi & -\frac{\varphi^2}{4}\sin 2\Psi & \varphi\cos\Psi\\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi\\ -\varphi\cos\Psi & -\varphi\sin\Psi & 1-\frac{\varphi^2}{2}\end{vmatrix}=\begin{vmatrix} -1+\frac{\varphi^2}{2}\cos^2\Psi & \frac{\varphi^2}{4}\sin 2\Psi & -\varphi\cos\Psi\\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi\\ \varphi\cos\Psi & \varphi\sin\Psi & -1+\frac{\varphi^2}{2}\end{vmatrix}. \quad (4.9)$$

Analogously, after the second turn the spin matrix is

$$M_{2/0}=\begin{vmatrix}-1&0&0\\0&1&0\\0&0&-1\end{vmatrix}\begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2(\theta+\Psi) & -\frac{\varphi^2}{4}\sin 2(\theta+\Psi) & \varphi\cos(\theta+\Psi)\\ -\frac{\varphi^2}{4}\sin 2(\theta+\Psi) & 1-\frac{\varphi^2}{2}\sin^2(\theta+\Psi) & \varphi\sin(\theta+\Psi)\\ -\varphi\cos(\theta+\Psi) & -\varphi\sin(\theta+\Psi) & 1-\frac{\varphi^2}{2}\end{vmatrix}$$

$$\times\begin{vmatrix} -1+\frac{\varphi^2}{2}\cos^2\Psi & \frac{\varphi^2}{4}\sin 2\Psi & -\varphi\cos\Psi\\ -\frac{\varphi^2}{4}\sin 2\Psi & 1-\frac{\varphi^2}{2}\sin^2\Psi & \varphi\sin\Psi\\ \varphi\cos\Psi & \varphi\sin\Psi & -1+\frac{\varphi^2}{2}\end{vmatrix}$$

$$=\begin{vmatrix} 1-\frac{\varphi^2}{2}[\cos\Psi+\cos(\theta+\Psi)]^2 & -\frac{\varphi^2}{4}[\sin 2\Psi-\sin 2(\theta+\Psi)+\cdots] & \varphi[\cos\Psi+\cos(\theta+\Psi)]\\ \frac{\varphi^2}{4}[-\sin 2\Psi+\sin 2(\theta+\Psi)+\cdots] & 1-\frac{\varphi^2}{2}[\sin\Psi-\sin(\theta+\Psi)]^2 & \varphi[\sin\Psi-\sin(\theta+\Psi)]\\ -\varphi[\cos\Psi+\cos(\theta+\Psi)] & -\varphi[\sin\Psi-\sin(\theta+\Psi)] & 1-\varphi^2-\varphi^2\cos(\theta+2\Psi)\end{vmatrix}. \quad (4.10)$$

Every element of the matrix (4.10) has a specific dependence on the turn number. Let us assume that after $(n+1)$ turns the resulting matrix of spin rotation will be as follows:

$$M_{(n+1)/0}=\begin{vmatrix} (-1)^{n+1}\left[1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n}\cos(i\theta+\Psi)\right)^2\right] & \frac{\varphi^2}{4}\left(\sum_{i=0}^{n}(-1)^{i+n}\sin 2(i\theta+\Psi)+\cdots\right) & (-1)^{n+1}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}\left(\sum_{i=0}^{n}(-1)^{i+1}\sin 2(i\theta+\Psi)+\cdots\right) & 1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi)\right)^2 & \varphi\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n}(-1)^{i+n}\sin(i\theta+\Psi) & (-1)^{n+1}\left(1-\frac{n+1}{2}\varphi^2+\cdots\right) \end{vmatrix}. \tag{4.11}$$

Then, multiplying the suggested matrix (4.11) by the matrix of spin advance in the next beam-beam interaction, Eq. (4.7), and by the matrix of spin advance in a ring, Eq. (4.1), the matrix after $(n+2)$ turns is obtained:

$$M_{(n+2)/0}=\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}\begin{vmatrix} 1-\frac{\varphi^2}{2}\cos^2[(n+1)\theta+\Psi] & -\frac{\varphi^2}{4}\sin 2[(n+1)\theta+\Psi] & \varphi\cos[(n+1)\theta+\Psi] \\ -\frac{\varphi^2}{4}\sin 2[(n+1)\theta+\Psi] & 1-\frac{\varphi^2}{2}\sin^2[(n+1)\theta+\Psi] & \varphi\sin[(n+1)\theta+\Psi] \\ -\varphi\cos[(n+1)\theta+\Psi] & -\varphi\sin[(n+1)\theta+\Psi] & 1-\frac{\varphi^2}{2} \end{vmatrix}$$

$$\times\begin{vmatrix} (-1)^{n+1}\left[1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n}\cos(i\theta+\Psi)\right)^2\right] & \frac{\varphi^2}{4}\left(\sum_{i=0}^{n}(-1)^{i+n}\sin 2(i\theta+\Psi)+\cdots\right) & (-1)^{n+1}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}\left(\sum_{i=0}^{n}(-1)^{i+1}\sin 2(i\theta+\Psi)+\cdots\right) & 1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi)\right)^2 & \varphi\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n}(-1)^{i+n}\sin(i\theta+\Psi) & (-1)^{n+1}\left(1-\frac{n+1}{2}\varphi^2+\cdots\right) \end{vmatrix}$$

$$=\begin{vmatrix} (-1)^{n+2}\left[1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n+1}\cos(i\theta+\Psi)\right)^2\right] & \frac{\varphi^2}{4}\left(\sum_{i=0}^{n+1}(-1)^{i+n+1}\sin 2(i\theta+\Psi)+\cdots\right) & (-1)^{n}\varphi\sum_{i=0}^{n+1}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}\left(\sum_{i=0}^{n+1}(-1)^{i+1}\sin 2(i\theta+\Psi)+\cdots\right) & 1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n+1}(-1)^{i}\sin(i\theta+\Psi)\right)^2 & \varphi\sum_{i=0}^{n+1}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n+1}\varphi\sum_{i=0}^{n+1}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n+1}(-1)^{i+n+1}\sin(i\theta+\Psi) & (-1)^{n+2}\left(1-\frac{n+2}{2}\varphi^2+\cdots\right) \end{vmatrix}. \tag{4.12}$$

The resulting matrix (4.12) can be written as the matrix (4.11), where the index (n) is substituted by the index $(n+1)$. Therefore, suggestion (4.11) is correct and gives the matrix of spin advance after an arbitrary number of turns.

C. Spin components after n turns

The developed approach gives us the possibility of predicting the effect of a beam-beam interaction on spin depolarization after a large number of turns. Suppose the initial spin vector has only one transverse component $S_y=1$ and the other components are equal to zero, $S_x=S_z=0$ (see Fig. 4). Spin advance is as follows:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix}=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}, \tag{4.13}$$

therefore only matrix elements a_{12},a_{22},a_{32} are essential to determine the values of spin components after n turns:

$$S_x=\frac{\varphi^2}{4}\left(\sum_{i=0}^{n-1}(-1)^{i+n-1}\sin 2(i\theta+\Psi)+\cdots\right)$$
$$=(-1)^{n-1}\frac{\varphi^2}{4}\frac{\sin\left(\frac{n(2\theta+\pi)}{2}\right)}{\cos\theta}$$
$$\times\sin\left(2\Psi+\frac{n-1}{2}(2\theta+\pi)\right)+\cdots, \tag{4.14}$$

$$S_y=1-\frac{\varphi^2}{2}\left(\sum_{i=0}^{n-1}(-1)^{i}\sin(i\theta+\Psi)\right)^2$$
$$=1-\frac{\varphi^2}{2}\frac{\sin^2\left(\frac{n(\theta+\pi)}{2}\right)}{\left(\cos\frac{\theta}{2}\right)^2}\sin^2\left(\Psi+\frac{n-1}{2}(\theta+\pi)\right), \tag{4.15}$$

$$S_z=(-1)^{n-1}\varphi\sum_{i=0}^{n-1}(-1)^i\sin(i\theta+\Psi)$$
$$=(-1)^{n-1}\varphi\,\frac{\sin\left(\frac{n(\theta+\pi)}{2}\right)}{\cos\frac{\theta}{2}}\sin\left(\Psi+\frac{n-1}{2}(\theta+\pi)\right). \tag{4.16}$$

The average values of spin components are achieved by integration of Eqs. (4.14)–(4.16) over all the initial phases:

$$\overline{S_x}=\frac{1}{2\pi}\int_0^{2\pi}S_x d\Psi=0, \tag{4.17}$$

$$\overline{S_y}=\frac{1}{2\pi}\int_0^{2\pi}S_y d\Psi=1-\frac{\varphi^2}{4}\,\frac{\sin^2\left(\frac{n(\theta+\pi)}{2}\right)}{\left(\cos\frac{\theta}{2}\right)^2}, \tag{4.18}$$

$$\overline{S_z}=\frac{1}{2\pi}\int_0^{2\pi}S_z d\Psi=0. \tag{4.19}$$

Mean-square values of spin components are given by

$$\langle S_x^2\rangle=\frac{1}{2\pi}\int_0^{2\pi}S_x^2 d\Psi=\frac{\varphi^4}{16}\left[\frac{\sin^2\left(\frac{n(2\theta+\pi)}{2}\right)}{2(\cos\theta)^2}+\cdots\right], \tag{4.20}$$

$$\langle S_y^2\rangle=\frac{1}{2\pi}\int_0^{2\pi}(S_y-\overline{S_y})^2 d\Psi=\frac{\varphi^4}{32}\,\frac{\sin^4\left(\frac{n(\theta+\pi)}{2}\right)}{\left(\cos\frac{\theta}{2}\right)^4}, \tag{4.21}$$

$$\langle S_z^2\rangle=\frac{1}{2\pi}\int_0^{2\pi}S_z^2 d\Psi=\frac{\varphi^2}{2}\,\frac{\sin^2\left(\frac{n(\theta+\pi)}{2}\right)}{\left(\cos\frac{\theta}{2}\right)^2}. \tag{4.22}$$

The introduced average and mean-square spin component parameters characterize spin depolarization. From formulas (4.17)–(4.22) it follows that they are turn dependent. Turn number n appears as an argument in trigonometric functions, providing oscillation of the average and mean-square spin parameters. Therefore, spin depolarization is suppressed. Taking the average values of the trigonometric functions

$$\overline{\sin^2\left(\frac{n(\theta+\pi)}{2}\right)}=\frac{1}{2}, \tag{4.23}$$

$$\overline{\sin^4\left(\frac{n(\theta+\pi)}{2}\right)}=\frac{3}{8}, \tag{4.24}$$

and the average values of the parameters φ,θ among all the particles,

$$\tilde{\varphi}\approx 4\pi G\gamma\xi\,\frac{\sigma}{\beta^*}, \tag{4.25}$$

$$\tilde{\theta}\approx 2\pi\left(Q-\frac{\xi}{2}\right), \tag{4.26}$$

the turn-independent average and mean-square spin parameters are

$$\overline{\overline{S_x}}=0,\quad \overline{\overline{S_y}}=1-\frac{\tilde{\varphi}^2}{8\left(\cos\frac{\tilde{\theta}}{2}\right)^2},\quad \overline{\overline{S_z}}=0, \tag{4.27}$$

$$\overline{\langle S_x^2\rangle}=\frac{\tilde{\varphi}^4}{16}\left(\frac{1}{4(\cos\tilde{\theta})^2}+\cdots\right),$$
$$\overline{\langle S_y^2\rangle}=\frac{3\tilde{\varphi}^4}{256\left(\cos\frac{\tilde{\theta}}{2}\right)^4}, \tag{4.28}$$
$$\overline{\langle S_z^2\rangle}=\frac{\tilde{\varphi}^2}{4\left(\cos\frac{\tilde{\theta}}{2}\right)^2}.$$

The attained formulas (4.27) and (4.28) indicate that spin depolarization due to beam-beam collisions is suppressed and depends on a betatron tune in a ring. The most dangerous working point is close to a half-integer value, because in that case the value of cos $\tilde{\theta}/2$ is close to zero and the spin depolarization parameters become large. The matrix (4.11) was obtained in a linear approximation to betatron particle motion and to beam-beam forces, therefore it cannot treat higher-order nonlinear spin resonances. Due to the small value of φ, depolarization effects, proportional to φ^4, are negligible as compared with those proportional to φ^2. Among possible depolarization effects, the most pronounced is a change of the values of $\overline{\overline{S_y}}$ and $\overline{\langle S_z^2\rangle}$.

V. NUMERICAL SIMULATION OF THE BEAM-BEAM EFFECT ON SPIN DEPOLARIZATION

A. Spin depolarization as a function of betatron tune

Computer simulations utilizing the numerical model of Sec. III were performed for the beam parameters, presented in Table II. For that combination of collider parameters, the values of the matrix parameters are as follows:

$$\tilde{\varphi}=4\pi G\gamma\xi\,\frac{\sigma}{\beta^*}=7.2\times10^{-3}, \tag{5.1}$$

$$\tilde{\theta}=2\pi(Q-0.006\,25). \tag{5.2}$$

Initial particle distribution in phase space was chosen to be Gaussian:

$$f=f_0\exp-\left(\frac{p_x^2+p_y^2}{2p_0^2}+\frac{x^2+y^2}{2\sigma^2}\right). \tag{5.3}$$

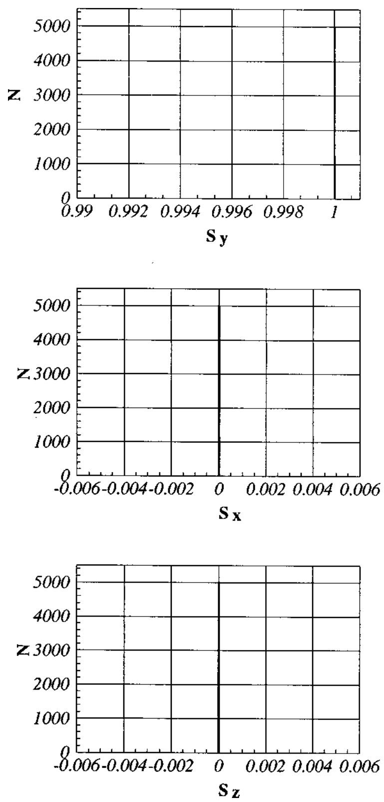

FIG. 4. Initial spin distribution.

During simulations, the average and rms values of spin parameters were calculated according to the formulas

$$\overline{S_x}=\frac{1}{N}\sum_{i=1}^{N} S_x(i), \tag{5.4a}$$

$$\overline{S_y}=\frac{1}{N}\sum_{i=1}^{N} S_y(i), \tag{5.4b}$$

$$\overline{S_z}=\frac{1}{N}\sum_{i=1}^{N} S_z(i), \tag{5.4c}$$

$$\sqrt{\langle S_x^2\rangle}=\left(\sum_{i=1}^{N}\frac{1}{N}[S_x(i)-\overline{S_x}]^2\right)^{1/2}, \tag{5.4d}$$

$$\sqrt{\langle S_y^2\rangle}=\left(\sum_{i=1}^{N}\frac{1}{N}[S_y(i)-\overline{S_y}]^2\right)^{1/2}, \tag{5.4e}$$

$$\sqrt{\langle S_z^2\rangle}=\left(\sum_{i=1}^{N}\frac{1}{N}[S_z(i)-\overline{S_z}]^2\right)^{1/2}. \tag{5.4f}$$

TABLE II. Parameters of the interacted beams.

Particle energy, γ	260
rms beam size at interaction point (IP), σ	0.08 mm
Beam-beam tune shift per collision ξ	-0.0125
Beta function, β^*	0.65 m

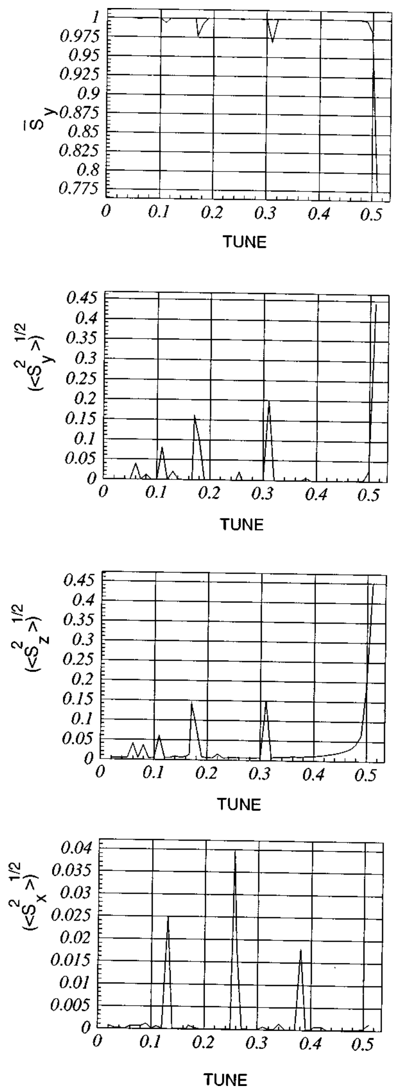

FIG. 5. Spin depolarization as a function of betatron tune.

In a storage ring, spin is a subject of intrinsic resonances, obeying the resonance condition

$$G\gamma=k_0+k_xQ_x+k_yQ_y, \tag{5.5}$$

where k_0, k_x, and k_y are integers. Average and rms spin components as a function of tune values are presented in Fig. 5. In that simulation, horizontal and vertical tunes were taken to be equal to each other, $Q_x=Q_y$. As seen, spin depolarization is most significant if the fractional part of the tune is close to 1/2, as was predicted by Eqs. (4.27) and (4.28). Also depolarization is observed if higher-order spin resonances are excited. Nonlinear spin resonances are not treated by analytical formulas of Sec. IV due to assumptions of the linear model. If tunes are far enough from that value, spin depolarization is suppressed.

In Figs. 6 and 7, results of suppressed spin depolarization for $Q_x=Q_y=14.43$ are presented. The average values of S_x and S_z are close to zero, as expected from Eqs. (4.27). The average value of S_y is slightly less than the initial value of 1, and oscillates around the stable value of 0.999 87. rms values of spin components are also oscillatory functions of turn number. Numerical values of average and rms values of spin

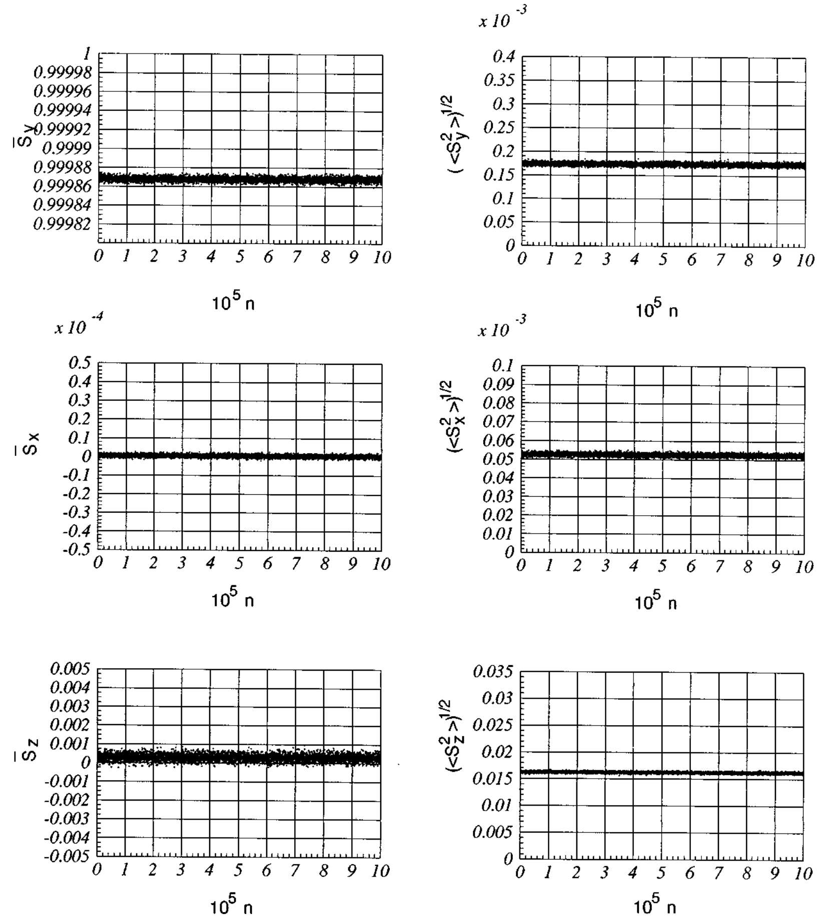

FIG. 6. Average and rms values of spin component as a function of turn number, $Q_x = Q_y = 14.43$.

components are close to analytical estimations (see Table III). Depolarization provides a small tail of distribution of the S_y component, which lasts from 1 to 0.9991. Distribution of the S_x component is much narrower than that of the S_z component. It also follows from Eqs. (4.28), where $\langle S_x^2 \rangle$ is proportional to φ^4, while $\langle S_z^2 \rangle$ is proportional to φ^2. Numerical simulations confirm the analytical prediction that spin depolarization due to the beam-beam interaction is suppressed if particle trajectories are stable and spin resonance conditions are avoided.

In Figs. 8 and 9, results of strong spin depolarization for $Q_x = Q_y = 14.505$ are presented. The average value of S_y is less than 0.5. rms values of S_y and S_z spin components are several orders of magnitude larger than for the previous case. Spin distribution has a spread from -1 to 1. It indicates significant depolarization, as expected from the results of the preceding section.

B. Spin depolarization in a ring without Siberian Snakes

To estimate the effect of Siberian Snakes on spin depolarization in the presence of a beam-beam interaction, consider a ring without Snakes. The derivation of spin matrix rotation after an arbitrary number of turns results in awkward expressions, so we have to rely on computer simulations. In Figs. 10 and 11, results of spin depolarization in a ring without Snakes are presented. Simulations were performed for the same values of betatron tunes $Q_x = Q_y = 14.43$ as in Figs.

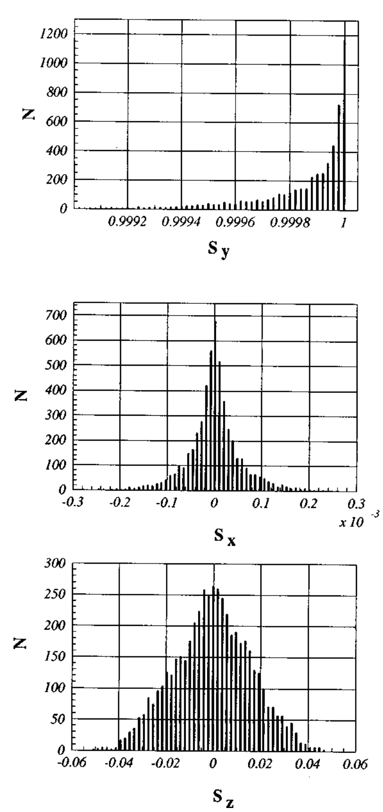

FIG. 7. Spin distribution after 10^6 turns, $Q_x = Q_y = 14.43$.

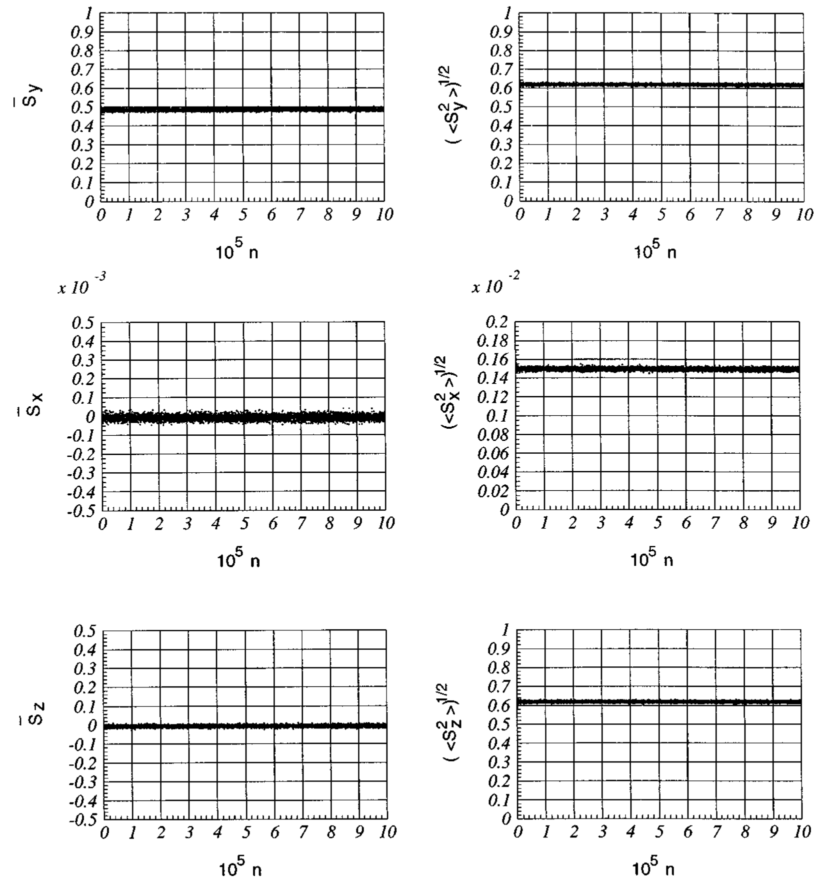

FIG. 8. Average and rms values of spin components as a function of turn number, $Q_x = Q_y = 14.505$.

6 and 7, where spin depolarization was suppressed. As seen, in the absence of Snakes, beam-beam collisions result in steady spin depolarization.

C. Spin depolarization in the presence of beam-beam instability

Up to now, we have considered particle motion in the presence of a stable beam-beam interaction. There are several mechanisms that lead to beam-beam instability. Excitation of nonlinear resonances and unstable stochastic particle motion due to overlapping of resonance islands is the fundamental phenomenon in beam-beam interaction [6]. Another mechanism of unstable particle motion is a diffusion created by random fluctuations in the distribution of the opposite

TABLE III. Average and rms spin components for $Q_x = Q_y = 14.43$.

	analytical	numerical
$\overline{S_y}$	0.999 88	0.999 88
$\overline{S_x}$	0	0
$\overline{S_z}$	0	0
$\sqrt{\langle S_y^2 \rangle}$	1.06×10^{-4}	1.7×10^{-4}
$\sqrt{\langle S_x^2 \rangle}$	7×10^{-6}	5×10^{-5}
$\sqrt{\langle S_z^2 \rangle}$	1.54×10^{-2}	1.6×10^{-2}

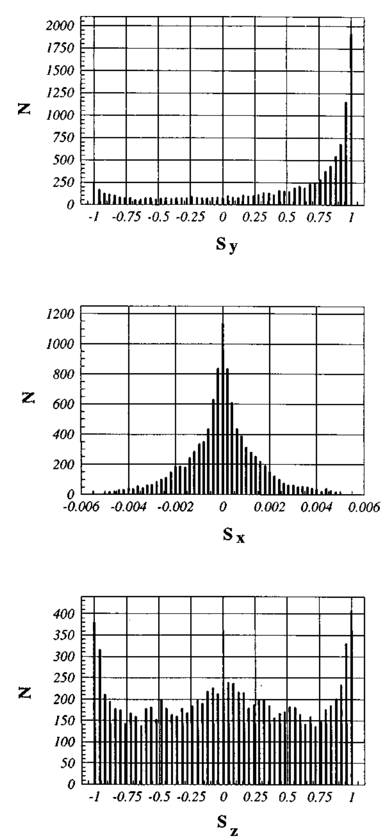

FIG. 9. Spin distribution after 10^6 turns, $Q_x = Q_y = 14.505$.

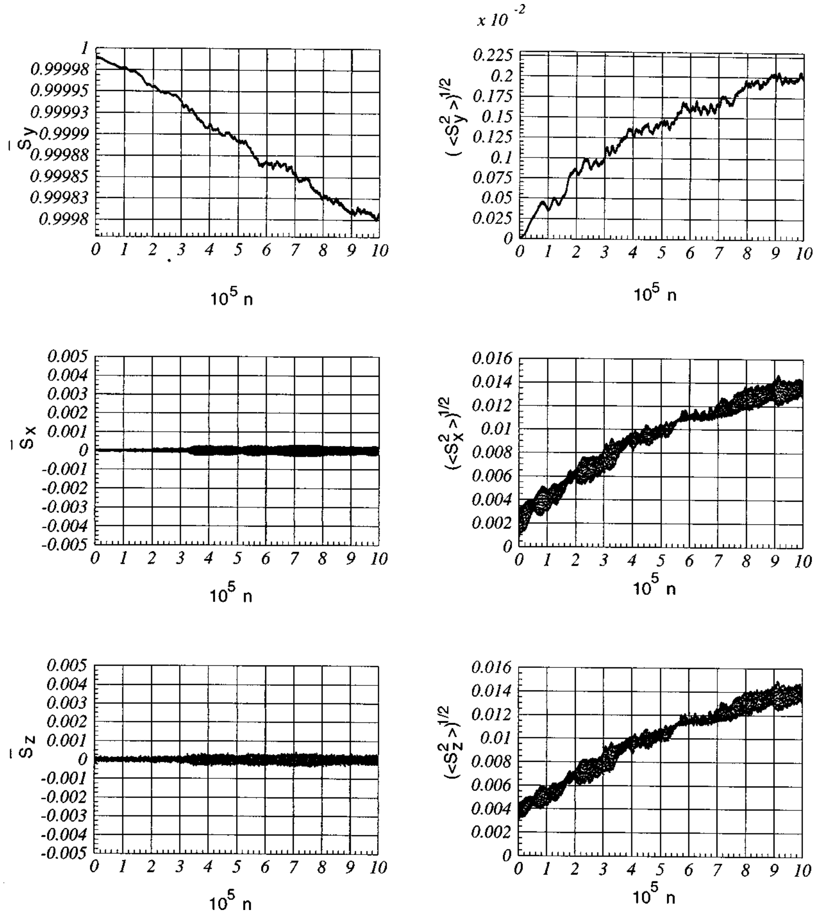

FIG. 10. Average and rms values of spin component as a function of turn number for a ring without Siberian Snakes, $Q_x=Q_y=14.43$.

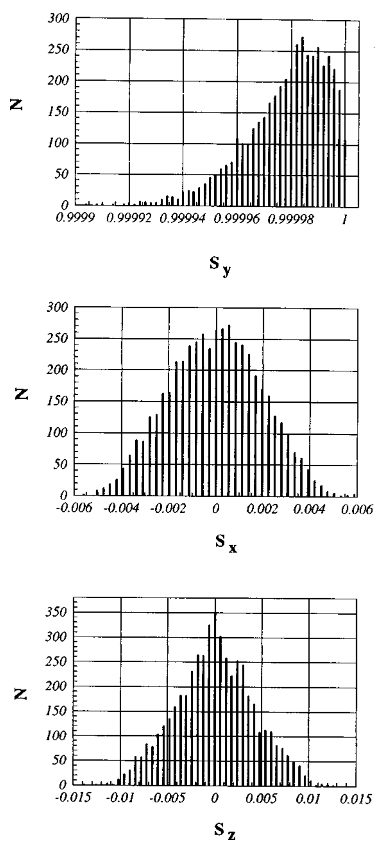

FIG. 11. Spin distribution after 10^6 turns in a ring without Siberian Snakes, $Q_x=Q_y=14.43$.

beam. In Ref. [7], noise beam-beam instability was studied for the case of random fluctuations in opposite beam size,

$$\sigma_n=\sigma_0\left(1\pm\frac{u(u_n)}{2}\right), \tag{5.6}$$

where u is a noise amplitude and u_n is a uniform random function with unit amplitude. It was shown that in the presence of noise, beam emittance is increased with time as

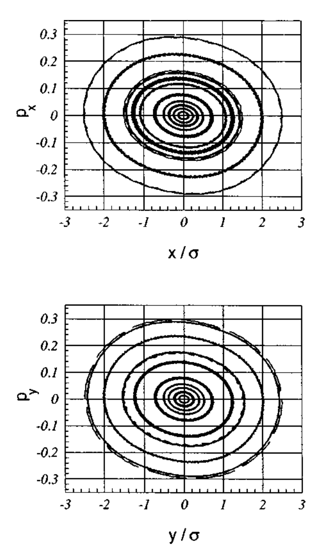

FIG. 12. Distorted particle trajectories in the presence of 2.5% noise in parameter σ in beam-beam kick, Eq. (3.2).

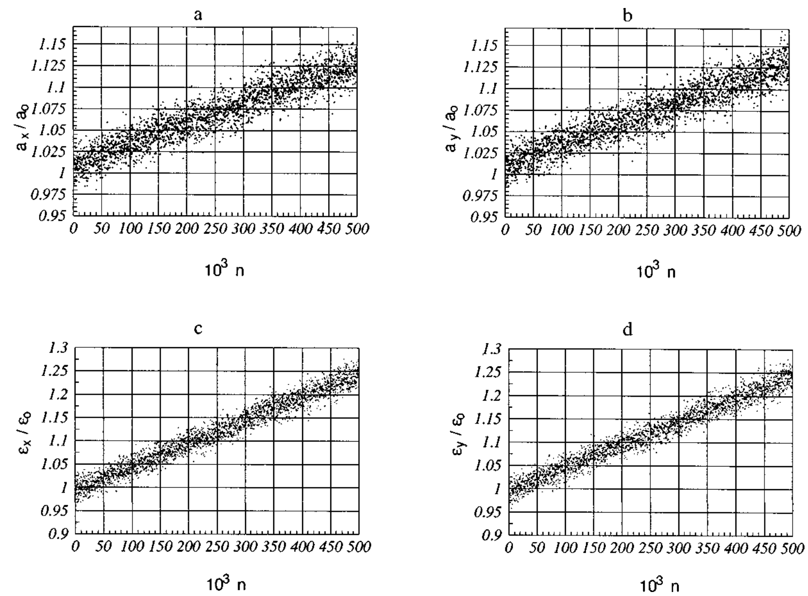

FIG. 13. (a) and (b) Beam envelopes; (c) and (d) beam emittances in the presence of a beam-beam interaction with 2.5% noise in parameter σ of beam-beam kick.

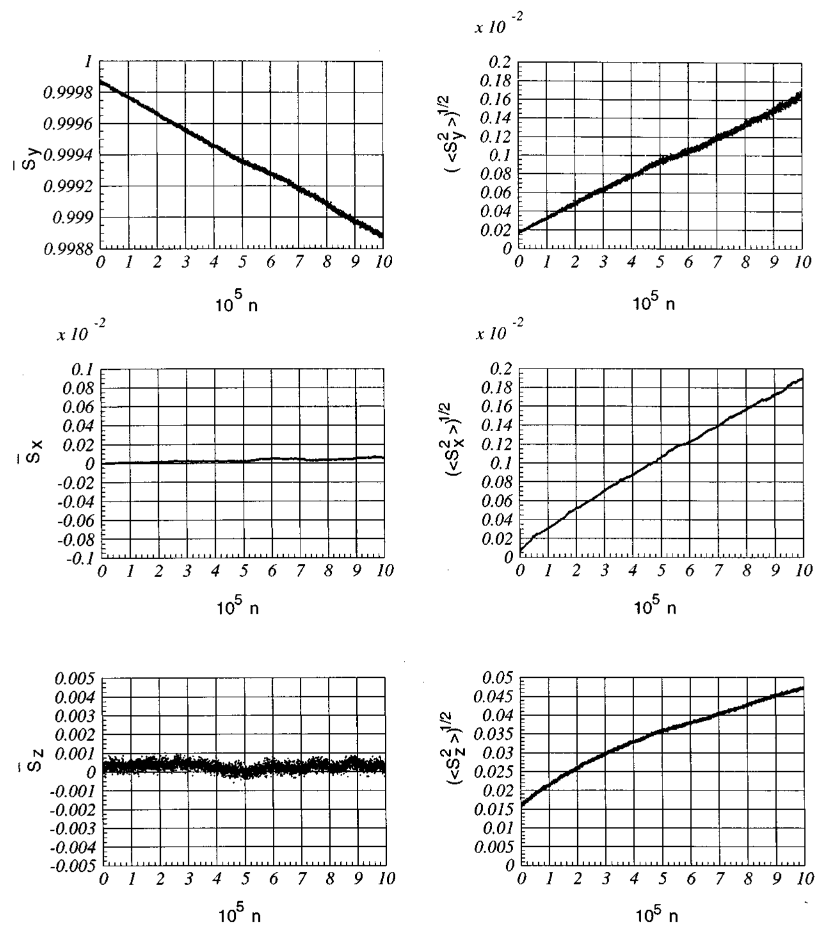

FIG. 14. Average and rms values of spin components as functions of turn number for a noisy beam-beam interaction, $Q_x = Q_y = 14.43$.

$$\frac{\varepsilon_n}{\varepsilon_0} = \sqrt{1+Dn}, \tag{5.7}$$

where the diffusion coefficient D is a function of beam-beam parameter ξ, noise amplitude u, and ratio of beam size, a, to opposite beam size, 2σ:

$$D = \pi^2 (\xi u)^2 \left(\frac{a}{2\sigma} \right)^4. \tag{5.8}$$

Noise in the beam-beam collision always induces instability if the beam-beam kick is a nonlinear function of the coordinate. Due to diffusion character, noise beam-beam instability does not have a threshold character and can exist at any value of the beam-beam parameter.

An increase of beam emittance is accompanied by an increase of beam size. In Figs. 12–14, results of a beam dynamics study and spin depolarization in the presence of a noisy beam-beam interaction are given. The value of noise amplitude $u = 0.025$ was chosen arbitrary, to demonstrate the main features of diffusion beam-beam instability. In contrast with Fig. 2, particle trajectories at phase planes are not closed (see Fig. 12). Beam emittances and beam envelopes are monotonous increasing functions of turn number (see Fig. 13). Increasing beam sizes results in steady spin depolarization (see Fig. 14). It is also expected from the analytical formulas (4.27) and (4.28), where the average and rms beam parameters are proportional to the powers of parameter φ, which, in turn, is proportional to the beam size according to Eq. (4.5). Therefore, beam-beam instability is a source of spin depolarization.

Spin depolarization due to beam-beam interaction was observed experimentally at the electron-positron collider PETRA [8]. Below the beam-beam limit, where particle motion was stable, spin depolarization was negligible. Above the beam-beam limit, a significant depolarization was observed, which was strongly correlated to beam blow up due to electron-positron collisions.

VI. CONCLUSIONS

The effect of beam-beam interaction on spin depolarization in a proton-proton collider has been studied. The employed method is based on a matrix formalism for spin advance and for perturbed betatron particle motion in a ring. Analytical calculations were done for a collider with one interaction point and two installed Siberian Snakes in each ring. A matrix for spin advance after an arbitrary number of turns is accomplished. The performed study indicates that spin depolarization due to beam-beam collisions is suppressed if beam-beam interaction is stable and spin resonances are avoided. Depolarization depends on the collider operation point. Unstable beam-beam interaction provides steady depolarization.

[1] V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Lett. **2**, 435 (1959).

[2] T. Katayama, RIKEN Internal Note, 1996 (unpublished).

[3] A. Chao, Nucl. Instrum. Methods **180**, 29 (1981).

[4] Ya. S. Derbenev, A. M. Kondratenko, S. I. Serednyakov, A. N. Skrinsky, G. M. Tumaikin, and Yu. M. Shatunov, Part. Accel. **8**, 115 (1978).

[5] Y. Batygin, in *Proceedings of the 3rd European Particle Accelerator Conference (EPAC92), Berlin, 1992*, edited by H. Henke, H. Homeyer, and Ch. Petit-Jean-Genaz (Fong and Sons, Singapore, 1992), p. 822.

[6] L. R. Evans, CERN Report No. 84-15, 319 (1984).

[7] Y. Batygin and T. Katayama, Nucl. Instrum. Methods Phys. Res. A **404**, 1 (1998).

[8] H. D. Bremer, J. Kewisch, H. C. Lewin, H. Mais, G. Ripken, R. Rossmanith, R. Schmidt, and D. P. Barber, DESY Report No. M-82-26 (1982).